Modeling and Control of PWM Fed 6-Phase Permanent Magnet Synchronous Machine

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Abstract—The paper presents a simulations study of a symmetrical and an asymmetrical six-phase permanent magnet synchronous machine (PMSM). The mathematical equations of considered models of the both system are described. Additional dynamic and steady states for Field Oriented Control (FOC) are simulated in Matlab/Simulink software.

I. INTRODUCTION

A power electronics has been utilized in many fields such a drives or an energy generation and conversion systems. Moreover, it is said that electric machines with a power electronic converters connected, are in their mature state now. This results in vast range of available control methods, different converter topologies and modulation techniques that are widely used over the industry sector. However, a continuous technology development involves these machine-converter sets more and more, even for high reliability applications where an ability for post-fault operation is demanded. Among them, the most fragile areas, are an aircraft energy generation system and off-shore wind farms. In the former case the safety is a number one objective. It is often ensured by utilization of redundant machines for providing continuous energy production ability. In case of remote systems such an off-shore wind farms, a fault tolerance is required due to maintenance and repair costs. Unfortunately a typical 3-phase machines despite their popularity are not the best solution for mentioned purposes. Machines are not simply able to continue their operation in case of any phase failure what often forces their disconnection. Mentioned issues lead for searching for a new solutions such a multiphase machines.

A Multi-phase machines have many advantages over a conventional 3-phase system as mentioned in [5]. One of them is a fact that for the same power rating a phase currents are much smaller in the multiphase system. The other beneficial properties are a reduction of electromagnetic torque ripples and higher power density coefficient. Due to a larger number of phases, multiphase machines are characterized by an inherent fault tolerance that can be improved. The application of the appropriate fault-tolerant algorithm reduces torque ripples [1, 2, 3, 4]. Note that many of typical 3-phase AC machines have have their multiphase counterpart with N phases (where N>3).

In high-power applications often machines with multiple three-phase windings are used. The most common case is a six-phase machine. There are two kinds of this machines [5]. The first is symmetrical machine in which stator windings are shifted by 60°. The most common is asymmetrical machine. In this kind of machine, the stator winding is composed of two 3-phase windings, which are spatially shifted by 30°. Larger number of asymmetrical motor application is due to the fact that design of the symmetrical motor causes more interference in the individual phases. The schemes for stator windings in asymmetrical and symmetrical machines are presented in Fig. 1.

![Fig.1. Stator windings in asymmetrical and symmetrical machine.](image)

However, the multiphase machines, despite their advantageous features are characterised by greater mathematical complexity. The most applications refer to multiphase induction motor (IM), but in this paper the synchronous machines (PMSM) are investigated due to higher efficiency, higher power factor, higher power density in comparison to IM for lower than 10 kW applications, resulting in smaller size and better heat transfer. Application of multiphase PMSM are included, among others, in papers [3, 6, 7-10].

This paper concentrates on modeling the both symmetrical and asymmetrical, 6-phase PMSM in healthy condition. Moreover a Field Oriented Control (FOC) [10-12] with PWM 3-level voltage converter is used to verify the machine model in simulation.

II. MATHEMATICAL DESCRIPTION OF 6-PHASE PMSM

An understanding of physical model of the machine is necessary for its proper control. Therefore, this section is devoted for mathematical description of mentioned motors.. Further in this paper a following simplifying assumptions are taken:

- Multi-phase stator winding shall be considered as concentrated winding.
- Motor magnetic circuit is linear.
- Parameters are temperature and time independent.

The basic equations that describe electric part of a synchronous machine in natural phase reference frame are shown below

\[ u_{si} = R_s i_{si} + \frac{d\psi_{si}}{dt} \quad (1) \]

for each phase i=1...6.

where the stator winding flux vector consists of rotor flux and stator flux linkages:
\[ \Psi_s = \Psi_s^{(r)} + \Psi_s^{(r)} \]  

(2)

The components of this equation are:

\[ \Psi_s^{(r)} = M I_s \]  

(3)

where \( M \) is matrix of self inductances and mutual inductances. Moreover, in case of permanent magnet motor a rotor flux equation can be described as (4)

\[ \Psi_s^{(r)} = \Psi_m F(\theta) \]  

(4)

where \(-\theta\) is electrical rotor position, \(\Psi_m\) is the amplitude of the flux linkages established by the permanent magnet on the rotor. \(F(\theta)\) is the vector function dependent on the construction of synchronous machine.

The equations described in stator reference frame could be simplified by transforming them into more natural frame. This frame rotates at the greatest angular speed equal to the angular frequency of the fundamental stator supply.

In this case equations of the 6-phase PMSM are [5]:

\[
\begin{align*}
    u_{ds} &= R_s i_{ds} + L_s \frac{di_{ds}}{dt} - \omega L_s i_{qs} \\
    u_{qs} &= R_s i_{qs} + L_s \frac{di_{qs}}{dt} + \omega L_s i_{ds} \\
    u_{x1s} &= R_s i_{x1s} + L_s \frac{di_{x1s}}{dt} \\
    u_{y1s} &= R_s i_{y1s} + L_s \frac{di_{y1s}}{dt} \\
    u_{0ps} &= R_s i_{0ps} + L_s \frac{di_{0ps}}{dt} \\
    u_{0ms} &= R_s i_{0ms} + L_s \frac{di_{0ms}}{dt}
\end{align*}
\]

(5)

The stator flux vector components are equal:

\[
\Psi_{ds} = L_s i_{ds} + \Psi_m \\
\Psi_{qs} = L_s i_{qs} \\
\Psi_{x1s} = L_s i_{x1s} \\
\Psi_{y1s} = L_s i_{y1s} \\
\Psi_{0ps} = L_s i_{0ps} \\
\Psi_{0ms} = L_s i_{0ms}
\]

(6)

where vectors of voltages, currents and fluxes in synchronous reference frame are defined appropriately as:

\[
\begin{align*}
    u_{dq} &= [u_{dq}, u_{ds}, u_{x1s}, u_{y1s}, u_{0ps}, u_{0ms}]^T \\
    i_{dq} &= [i_{dq}, i_{ds}, i_{x1s}, i_{y1s}, i_{0ps}, i_{0ms}]^T \\
    \Psi_{dq} &= [\Psi_{dq}, \Psi_{ds}, \Psi_{x1s}, \Psi_{y1s}, \Psi_{0ps}, \Psi_{0ms}]^T
\end{align*}
\]

Additionally, the electromagnetic torque is equal:

\[ m_e = p \Psi_m i_{qs} \]  

and the mechanical equation is expressed as:

\[ J \frac{d\omega_m}{dt} = (m_e - m_i) \]  

(9)

In order to change the phase system to the synchronous dq system must perform two transformations. The first transformation is performed with the phase (ph) to stationary (\(\alpha, \beta\)) system for any vector (current, voltage, flux). This vector is called \(\text{vec}\) by convention:

\[ \text{vec}_{ab} = T \text{vec}_{ph} \]  

(10)

The mathematical models were shown for symmetrical and asymmetrical machines. The matrix of the transformation \(T\) for the symmetrical model has the following form:

\[
T = \begin{bmatrix}
1 & \cos(y) & \cos(2y) & \cos(3y) & \cos(4y) & \cos(5y) \\
0 & \sin(y) & \sin(2y) & \sin(3y) & \sin(4y) & \sin(5y) \\
1 & \cos(2y) & \cos(4y) & \cos(6y) & \cos(8y) & \cos(10y) \\
0 & \sin(2y) & \sin(4y) & \sin(6y) & \sin(8y) & \sin(10y) \\
1 & -\sin(y) & \sin(2y) & -\sin(3y) & \sin(4y) & -\sin(5y) \\
1 & -\sin(2y) & \sin(4y) & -\sin(6y) & \sin(8y) & -\sin(10y)
\end{bmatrix}
\]

(11)

where \(y = \pi/3\).

The transformation matrix \(T\) for asymmetrical six-phase machine is of the form

\[
T = \begin{bmatrix}
1 & \cos(\pi/6) & \cos(2\pi/3) & \cos(5\pi/6) & \cos(4\pi/3) & \cos(3\pi/2) \\
0 & \sin(\pi/6) & \sin(2\pi/3) & \sin(5\pi/6) & \sin(4\pi/3) & \sin(3\pi/2) \\
1 & \cos(5\pi/6) & \cos(4\pi/3) & \cos(3\pi/2) & \cos(2\pi/3) & \cos(\pi/6) \\
0 & \sin(5\pi/6) & \sin(4\pi/3) & \sin(3\pi/2) & \sin(2\pi/3) & \sin(\pi/6) \\
1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0
\end{bmatrix}
\]

(12)

The second transformation, allows for transition from the \(\alpha\beta\) to the synchronous \(dq\) reference frame.

\[ \text{vec}_{dq} = D \text{vec}_{ab} \]  

(13)

Matrix \(D\) could be described as follows:

\[
D = \begin{bmatrix}
\cos(\theta) & \sin(\theta) & 0 & 0 & 0 & 0 \\
-\sin(\theta) & \cos(\theta) & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

(14)

\[ \theta \] is an angle of the synchronous system with reference to the stator system, and is expressed in following form:

\[ \theta = \int \omega dt \]  

(15)

and \(\omega\) is the synchronous speed.

Using the inverse transformation i.e. from the \(dq\) system to phase system two transformations are used. First from them transforms the \(dq\) system to \(\alpha\beta\).

\[ \text{vec}_{ab} = D^T \text{vec}_{dq} \]  

(16)

It should be noted, that the matrices \(T\) and \(D\) are orthogonal, so the inverse of any of this matrixes are equal to transposition of them. This observation causes, that calculation of inverse matrices are much easier.

Second transformation from stationary \(\alpha\beta\) system to phase system is equal:

\[ \text{vec}_{ph} = T^T \text{vec}_{ab} \]  

(17)
Using above formulas it is possible to build the model of a multi-phase synchronous motor in Simulink. The electrical part of PMSM scheme in Simulink is presented in Fig. 2.

![Fig. 2. The electrical part of PMSM block for Simulink simulation.](image)

**III. FOC SYSTEM IMPLEMENTATION FOR 6-PASE PMSM**

In order to assure correct work of the system in dynamic and steady states, one should use the control method with speed tracking. The FOC [10-12] method is used to control 6-phase PMSM. During healthy operation only d, q components take part in electromagnetic torque generation and FOC scheme is identical as it is for 3-phase machine. It can be noticed that \( i_f \) component corresponds to flux and \( i_q \) for torque production respectively. So, the reference value of current controller in the q axis was setting to the output value of the speed controller. For the speed control as the input was set the desired speed of the system. In order to regulate the flux was used current regulator in the d axis, which value was setting to zero. Additionally, decoupling in current controllers with SEM signals, is applied. The block scheme of FOC system for multiphase PMSM is presented in Fig 3.

![Fig. 3. FOC system for multiphase PMSM.](image)

This method was modeled in Simulink programs. Block scheme of FOC system in Simulink is presented in Fig. 4.

![Fig. 4. Block scheme of FOC system in Simulink.](image)

**IV. SIMULATION RESULTS**

In this section the simulation results of FOC for healthy multiphase machine operation, is addressed. Necessary data used in simulation is presented in **TABLE I**.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetic flux-linkage</td>
<td>( \psi_{ss} )</td>
<td>0.610 Wb</td>
</tr>
<tr>
<td>Stator inductance</td>
<td>( L_s )</td>
<td>40 mH</td>
</tr>
<tr>
<td>Stator resistance</td>
<td>( R_s )</td>
<td>3 ( \Omega )</td>
</tr>
<tr>
<td>Moment of the inertia</td>
<td>( J )</td>
<td>0.02 kg/m(^2)</td>
</tr>
<tr>
<td>Number of pole pairs</td>
<td>( p )</td>
<td>2</td>
</tr>
<tr>
<td>Mutual stator inductance</td>
<td>( L_{1s} )</td>
<td>4 mH</td>
</tr>
</tbody>
</table>

Fig. 5. and Fig. 6. presents the behavior of 6-phase synchronous machine in dynamical states, where Fig 5. shows the response to load torque step change and Fig 6. the response to load speed step change. It could be noted that system works with 3-level modulator. In Fig. 7. the difference between the symmetrical and asymmetrical machine model in the steady states condition is shown.

**V. CONCLUSIONS**

The problems of 6-phase permanent magnet synchronous machine modeling are discussed. The transformation of machine phase variables into new coordinate system is presented. The equations of 6-phase synchronous symmetrical and asymmetrical machine for transformed coordinates are given and discussed. The methods of field-oriented vector control of multi-phase synchronous machine are presented. The results of simulations of field-oriented vector control of PWM converter fed 6-phase Permanent Magnet Synchronous Machine are presented in steady and dynamical states. These simulations were performed with using Simulink/Matlab program.

**ACKNOWLEDGMENT**

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Fig. 5. Dynamical state for 6-phase asymmetrical PMSM. Response to load torque step change. From the top: 1) speed and reference speed, 2) electromagnetic torque and load torque, 3) phase voltage, 4) line-to-line voltage 5) reference voltages from FOC system, 6) phase current.

Fig. 6. Dynamical state for 6-phase symmetrical PMSM. Response to speed step change. From the top: 1) speed and reference speed, 2) electromagnetic torque and load torque, 3) phase voltage, 4) line-to-line voltage 5) reference voltages from FOC system, 6) phase current.

Rys 7. Steady state for 6-phase PMSM. Phase currents and reference voltages from FOC system. Symmetrical and asymmetrical model.

REFERENCES


