Abstract The paper presents the idea of the transportation task optimization by selecting the best method of the control functions calculation, with application of the modern hardware-software means.

The paper discusses general principles of model building by means of mathematical programming. The paper suggests formulations and solutions for practical problems. The individual moments of the methodology for the synthesis of controller organization process are discussed in this article. The simulation results using algorithms and draw conclusions about the possibility of creating a discrete model for the optimal control are proposed.

Keywords: Transportation of goods, industrial controller (PLC), model, SCADA system.

I. INTRODUCTION

Mathematical simulation or mathematical modeling is a mathematical representation of real objects, processes and phenomena and study them using this representation.

The study of objects with the help of models is carried out by the methods of the operations research, part of which can be considered as mathematical programming and its sub—linear programming. Following the general optimization paradigm (Fig.1), when applying operations research, a decision-maker selects the key decision variables that will influence the overall quality of decisions. Quality of decisions is expressed by the objective function that is maximized (profit, product quality, or job completion), or minimized (cost, loss, risk of some undesirable event). In addition to the objective function, a set of limitations is also considered. Adjusting the values of all decision variables, optimal solution is selected in the context of the given problem (model) formulation [1].

The development of any such project and modeling studies can be roughly divided into six steps:

• Formulation of the problem
• Development of a mathematical model of the problem being studied
  • Finding solutions with the help of this model
  • Verification of the model and solutions
  • Clarifying the decision
  • Application of solutions in practice

When working on the problem it is necessary to describe the object using variables, to build a model that will connect to each other variables, define limitations for variables and responses to the object in terms of realism and efficiency [2].

It is necessary to find a solution to optimize it and, most importantly, find the model developed application for practical use, to evaluate the behavior of the solution in a real environment. In terms of linear programming mathematical model is a set of relations between variables, resources, limitations and the objective function (measure of efficiency) [3].

Modeling is also the creation of the highest possible replicas of real physical objects (buildings, ships, planes, trains). The use of modern electronics in the models allows doing more and more “smart” models, increasingly approximating their behavior to the behavior of those real objects which these models have copied.

Control system for railroad model (Fig.2) allows to simulate the transportation operation of homogeneous goods and to find the optimal routes.

The following tasks are required for this:

• to implement the decision by the selection of equipment, to carry out necessary calculations, to write the control program;
• to develop algorithms for solving the transportation problem of linear programming and to write code for implementation in the selected system;
II Mathematical Model of a Transportation Problem

Suppose that there are \( m \) points of the homogeneous goods departure \( A1, A2, A3, \ldots, Am \) and \( n \) shipping dates of acceptance of \( B1, B2, B3, \ldots, Bn \).

Goods can be delivered between any two points. In this case, \( ai > 0 \) - stock goods at the point \( Ai \), \( bj > 0 \) - stock goods at \( Bj \) and the transportation cost from \( Ai \) to \( Bj \) \( cij \geq 0 \).

It is required to carry out the goods from the departure points to the destination points, under the condition, that the transportation cost is minimal. The mathematical model of the problem is called the transportation problem \([4]\). It consists of the following:

Find a non-negative matrix \( X \), satisfying the conditions:

\[
\sum a_{i}x_{i,j} = b_{j} \quad (i = 1 \text{ to } n; j = 1 \text{ to } m),
\]

when the function minimum is:

\[
L(X) = \sum c_{ij}x_{ij} \quad (i = 1 \text{ to } n; j = 1 \text{ to } m),
\]

where \( c_{ij}x_{ij} \) - the cost of moving \( x_{ij} \) units of goods from point \( Ai \) to point \( Bj \).

A. The main theorems of the transportation problem

Theorem 1. Any transportation problem, in which the amount of reserves equal to the amount of requirements, has a solution.

Theorem 2. If the transportation problem planning matrix \( X = (xi,j) \) is an optimal, then it corresponds to a system of \((m + n)\) numbers \( U_{i}^{*} \) and \( V_{j}^{*} \), satisfying the following conditions:

\[
U_{i}^{*} + V_{j}^{*} = c_{ij} \quad \text{for} \quad x_{i,j} > 0 \quad (1)
\]

\[
U_{i}^{*} + V_{j}^{*} \leq c_{ij} \quad \text{for} \quad x_{i,j} = 0 \quad (2)
\]

Numbers \( U_{i}^{*} \) and \( V_{j}^{*} \) are called the potentials of the goods departure points and destination points respectively. So, in order to the reference program has been the best, it is necessary that for each occupied cell the sum of potentials would be equal to the sum of the transportation unit cost, and for the unoccupied - is less than or equal to the transportation unit cost for a given cell.

Theorem 3. For any unoccupied cell of planning matrix a cycle can be built \([5]\).

B. Basic steps of solution

The solution of this transportation problem can be divided into three steps:

- Preparation of planning matrix and the initial reference plan finding by North West corner rule;
- Writing equations for the potentials of unoccupied cells and finding these potentials;
- Verification of solutions optimality and if it is not optimal, drafting cycle for recalculation and repeating all the steps again \([6]\).

To calculate the optimal route of goods between the stations on the developed railroad model the industrial controller is used. A user can enter the initial data (the cost of transportation and the number goods), and the controller program calculates routes and gives the result. Calculation algorithms can be programmed in different programming languages, but the first program is created in pseudo-code, which can then be easily placed in one of the programming languages of a specific project.

This allows to realize the problem for the different types of controllers, that is, to achieve the solutions portability and versatility, compatibility between the physical model and the different industrial equipment.

C. Features of Pseudo-code

Different applications may include any number of the storage stations and destination stations (within the limitations of the total number of stations). That is, there may be a combination of the following stations:

\[
2 \times 6, \ 3 \times 5, \ 4 \times 4, \ 5 \times 3, \ 6 \times 2.
\]

Considering that the quantity of the goods to the destination may not correspond to the quantity the goods at points of acceptance, it is necessary to provide another row or another column for the planning matrix in order to add destination or storage stations.

Possibilities of PLC programming languages are limited compared to high level programming languages. These limitations are taken into account while writing pseudo-code to would be able to transfer this code with minimal changes in the maximum possible number of programming environments \([7]\).

III. Solution of the Transportation Problem

A. Preparation of Planning Matrix

The first step in solving the problem - finding the optimal route of the goods – construction of the table, called the planning matrix and finding the initial reference program.

If the number of the goods in warehouses does not correspond to the needs (a problem is open), one more station is added, the transportation cost at which is considered to be zero. If the amount of the goods more than required, another dummy destination station is added with zero cost traffic.

Need for the goods (or the amount of the goods) on this the dummy station is considered equal to the excess of (or lack of). Then a number of warehouses (taken in any order) is built into the column. Station customers (in random order) are arranged in a row. The row stations are numbered. The table compiled from data row and column of the matrix is the basis of the planning matrix (Fig.3).

![Planning matrix, reference program drafting.](image)

B. Problem solution by North West corner rule

As a result, the table cells with the goods, called occupied, and the empty cells - unoccupied are formed.

Figure 3 shows the goods distribution by the north-west corner method.

This planning matrix may not be optimal. To estimate the transportation cost by such distribution the objective function is used \([8]\):

\[
z = c_{1,4} \times z_{1,4} + c_{1,5} \times z_{1,5} + c_{2,2} \times z_{2,2} + c_{2,3} \times z_{2,3} + c_{2,5} \times z_{2,5} + c_{3,1} \times z_{3,1} + c_{3,5} \times z_{3,5},
\]

where \( c_{1,4} \) is the cost of the goods transportation from...
station \( A_i \) to station \( B_j \), and \( z_{1,4} \) is the number of the goods units during transportation between these stations.

Algorithm for the problem solving in pseudo-code is the following (Fig.4):

```c
if(a < 0) {
    maxValues[i][j] = a; /\* (1) \*/
    r = (-1)*a;
    i++;}
for(a = 1; a <= 9; a++)
  if(maxValues[i] == 1)
    a = maxValues[a];
break;
}
```

Fig. 4. Pseudo-code part

C. Potentials Finding

The way to solve the transportation problem by the potential method was published by L. Kantorovich [6].

So, let \( U_i \) and \( V_j \) are potentials \( i \)-th warehouse and the \( j \)-th customer, respectively. Then the equations are true in the form:

\[
U_i + V_j = C_{i,j},
\]

where \( C_{i,j} \) - the sum of the potentials in the occupied cells. Thus, since the occupied cells \((m+n-1)\), a system of \((m+n-1)\) equations can be created with \((n+m-2)\) variables.

Based on the previous completed table the following equation solution looks like this:

\[
\begin{align*}
S_{1,1} &= C_{1,1} - (U_1 + V_1) = -2 \\
S_{1,2} &= C_{1,2} - (U_1 + V_2) = 13 \\
S_{1,3} &= C_{1,3} - (U_1 + V_3) = 9 \\
S_{2,1} &= C_{2,1} - (U_2 + V_1) = 2 \\
S_{2,4} &= C_{2,4} - (U_2 + V_4) = -1 \\
S_{3,2} &= C_{3,2} - (U_3 + V_2) = 17 \\
S_{3,3} &= C_{3,3} - (U_3 + V_3) = 15 \\
S_{3,4} &= C_{3,4} - (U_3 + V_4) = 10
\end{align*}
\]

It is clearly seen that two values of the differences are negative. It means that planning matrix is non-optimal. To optimize planning matrix a cycle recalculation is required to be built.

B. Formation of the Recalculation Cycle

The cycle is a few cells, connected of a closed broken line in each cell turning at 90°. It can always be built according to Theorem 3. As a result of this search algorithm for the planning matrix is found, and it exists always by Theorem 1 (Fig.5).

<table>
<thead>
<tr>
<th>Station-warehouse</th>
<th>Station-customer</th>
<th>Goods</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>A2</td>
<td>A3</td>
</tr>
<tr>
<td>B1</td>
<td>29</td>
<td>15</td>
</tr>
<tr>
<td>B2</td>
<td>23</td>
<td>19</td>
</tr>
<tr>
<td>B3</td>
<td>29</td>
<td>16</td>
</tr>
<tr>
<td>B4</td>
<td>26</td>
<td>10</td>
</tr>
<tr>
<td>B5</td>
<td>230</td>
<td>110</td>
</tr>
</tbody>
</table>

IV. CONTROL SYSTEM CONFIGURATION

PLC ABB AC500 family and expansion unit DA501 are used to control. The controller has a built-in WEB-server that permits to access the process by means of a web browser (Fig.6). ABB IP Configuration allows finding all the devices firm ABB, connected to the network (by MAC-address).

The control development environment is Control Builder Plus software (Fig.7).

CoDeSys is the software for creating the code for controller, as well as for the visualization creating. The program supports five IEC61131-3 standard programming languages [9]. Main program block PLC_PRG is shown on Fig.8.

V. RESULTS

Control system for the railroad model is completely designed via visualization system (SCADA-system). Visualization is created in the same environment CoDeSys, as the code for the
controller because of Control Builder Plus is an integrated programming environment (Fig. 9 - 11).

Fig. 9. The number of warehouses (blue) and the goods designation.

Fig. 10. Cost of traffic calculation.

Fig. 11. The beginning of the train movement (green arc).

In Figures green stations are the warehouses, red stations are the destination stations. Yellow squares indicate the location of the sensors, reed switches. At the same time it means that railway track is divided into separate areas, where the train position is determined by the positioning system.

CONCLUSION

A mathematical model to find the best ways of homogeneous goods transportation between the railway stations is developed. An algorithm for the optimal routes transportation is written, using methods of mathematical programming. Control program and system of visualization are created for the industrial controller. The railroad model is made based on a combination of the finished industrial simulation kits (the scale of 1: 87) with industrial equipment from ABB, as well as with self-made electronic components and parts, using construction and decorative materials.

The emergency stop problem in the case of impossibility to pass along the railway (divorced arrow) is implemented. Created control project for railway visualization with a choice of the train route has the ability to set the number of passing trains on all routes, the ability to choose warehouses and destination stations for the goods transportation.

In the future it is possible to build a model using more than one composition, to calculate the optimal movement of the different goods types to different stations by different trains. The control system allows monitor the motion safety, avoiding possible collision of the trains.

REFERENCES


