Parameters Identification of Equivalent Circuit Diagrams for Li-Ion Batteries

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***Abstract*-Batteries play an essential role in electric vehicles (EV), and obtain more and more importance also in smart grids due to the non-constant power generation of renewable energy sources. In order to achieve an optimum operation of systems with batteries it is necessary to develop accurate mathematical models for the calculation of the state of charge (SOC), the temperature distribution within the battery, the residual capacity, the internal resistance and the lifetime, taking into account the individual operation by the user. This paper presents the fundamentals of a method how to determine SOC of lithium-ion batteries on the basis of two different equivalent circuit diagrams and an Extended Kalman Filter (EKF). It is described how to identify the parameters of these circuits by characteristic measurements. The comparison between measurement and computation results shows a good accordance. The accurate determination of SOC of a battery in an EV is of high importance for the prediction of the distance that can be driven. In the first step the dependency of these parameters on the temperature and on the battery age is neglected.**

I. INTRODUCTION

If a defined full charge of a battery takes place regularly it is possible to determine the state of charge (SOC) by the so-called Ampere-counting method. This method is basing on the charge that is transferred into the battery respectively taken out of the battery. In case where a defined full charge of the battery cannot happen regularly the error in the SOC estimation can become inacceptable high and a better method has to be found. The SOC is a function of the open circuit voltage (OCV) of a lithium-ion battery, $SOC=f(V_{OC})$, but this method involves the problem of its dynamics as Fig. 3c and 3d demonstrate. The electrochemical processes which take place in a cell result in the fact that the OCV cannot be measured at the battery terminals. The dynamics needs to be modeled mathematically in a way that the OCV respectively SOC can be calculated by measuring only the battery voltage and current at the terminals of the battery. For this purpose an equivalent circuit diagram for the battery cell has to be used, and the parameters need to be identified by characteristic measurements.

In this paper at first the internal resistance model (IR) is presented, then the one time constant model (OTC), and finally the two time constants model (TTC). Further, comparisons between the model-based simulation data and the measured data are carried out to evaluate the validity of the demonstrated models, which provide a foundation for the model based SOC estimation [4].

II. EQUIVALENT CIRCUIT MODELS

A. IR Model:

The IR model as shown in Fig. 1a, and described by (1) implements an ideal voltage source V_{OC} that represents the open-circuit voltage (OCV) of the battery, and an ohmic resistance in order to describe the internal resistance. Both, resistance and open-circuit voltage V_{OC} are functions of SOC, state of health (SOH) and temperature. i_{Batt} is the battery output current with a positive value when discharging, and a negative value when charging, v_{Batt} is the battery terminal voltage [1].

$$v_{Batt} = V_{OC} - R_0 * i_{Batt} \,. \tag{1}$$

As the IR model does not represent the transient behavior of lithium-ion cells, it is not suitable for the accurate estimation of SOC during any dynamical operation (non-constant load).

B. OTC Model:

The OTC model adds a parallel RC network in series to the internal resistance R_0 of the IR model, in order to approximate the dynamic behaviour of the lithium-ion battery. As shown in Fig. 1b, it mainly consists of three parts including the voltage source V_{OC}, the ohmic resistance R_0, and R_{OTC},C_{OTC} to describe the battery transient response during charging or discharging. v_{OTC} is the voltage across C_{OTC}; i_{OTC} is the current that flows in C_{OTC}. The electric behaviour of the OTC model can be expressed by (2) and (3) in continuous time [1]:

$$\dot{v}_{OTC} = -1/(R_{OTC} * C_{OTC}) * v_{OTC} + 1/(C_{OTC}) * i_{Batt} \,, \tag{2}$$

$$v_{Batt} = V_{OC} - v_{OTC} - R_0 * i_{Batt} \,. \tag{3}$$

The description in discrete time is shown by (4) and (5), where T_S is the sampling period.

$$\begin{aligned} v_{OTC,k+1} = {} & v_{OTC,k} * \exp[-T_S /(R_{OTC} * C_{OTC})] + \\ & R_{OTC} * \{1 - \exp[-T_S /(R_{OTC} * C_{OTC})]\} * i_{Batt,k} \end{aligned} \,, \tag{4}$$

$$v_{Batt,k} = V_{OC}(SOC_k) - v_{OTC,k} - R_0 * i_{Batt,k} \,. \tag{5}$$

C. TTC Model:

Basing on the observation of the battery output voltage when the battery output current is zero (no-load) it has been found out that the battery shows a big difference between the short time and the long time transient behavior. That means the dynamic characteristics cannot be represented very accurately by the OTC model.

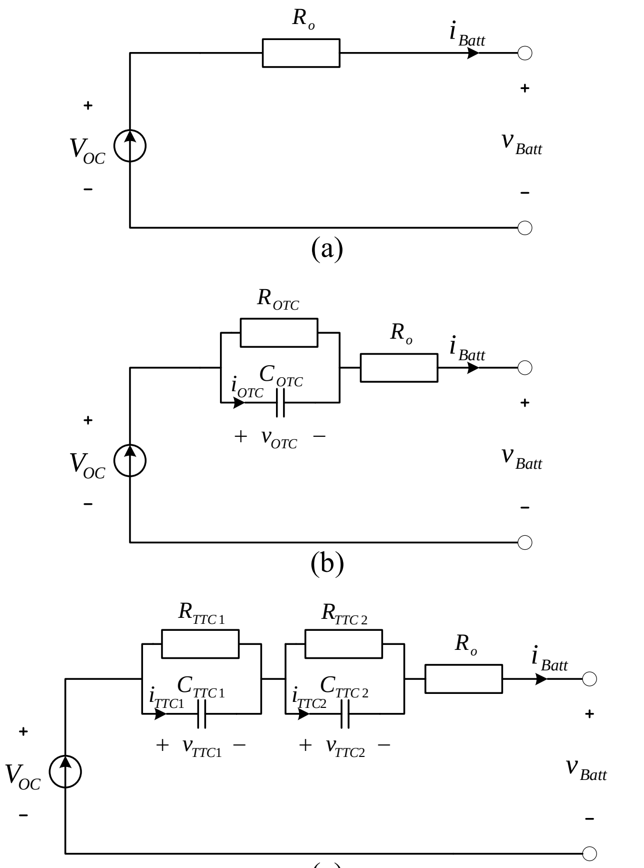

Fig. 1. Battery equivalent circuit diagrams. (a): internal resistance model (IR); (b): one time constant model (OTC); (c): two time constants model (TTC).

To improve the flexibility of the OTC model an extra RC network is added in series to the OTC circuit to get the TTC circuit model. As shown in Fig. 1c, the TTC circuit is composed of four parts: voltage source V_{OC}, ohmic resistance R_0, R_{TTC1} and C_{TTC1} to describe the short term characteristics, R_{TTC2} and C_{TTC2} to describe the long term characteristics. v_{TTC1} and v_{TTC2} are the voltages across C_{TTC1} and C_{TTC2} respectively. i_{TTC1} and i_{TTC2} are the outflow currents of C_{TTC1} and C_{TTC2} respectively [1].

The electrical behavior of the TTC circuit can be expressed by (6), (7) and (8) in continuous time:

$$\dot{v}_{TTC1} = -1/(C_{TTC1} * R_{TTC1}) * v_{TTC1} + 1/(C_{TTC1}) * i_{Batt}\,, \quad (6)$$

$$\dot{v}_{TTC2} = -1/(C_{TTC2} * R_{TTC2}) * v_{TTC2} + 1/(C_{TTC2}) * i_{Batt}\,, \quad (7)$$

$$v_{Batt} = V_{OC} - v_{TTC1} - v_{TTC2} - R_0 * i_{Batt}\,. \quad (8)$$

The description in discrete time is given by (9), (10) and (11):

$$v_{TTC1,k+1} = v_{TTC1,k} * \exp[-T_S/(R_{TTC1} * C_{TTC1})] + R_{TTC1} * \{1 - \exp[-T_S/(R_{TTC1} * C_{TTC1})]\} * i_{Batt,k}\,, \quad (9)$$

$$v_{TTC2,k+1} = v_{TTC2,k} * \exp[-T_S/(R_{TTC2} * C_{TTC2})] + R_{TTC2} * \{1 - \exp[-T_S/(R_{TTC2} * C_{TTC2})]\} * i_{Batt,k}\,, \quad (10)$$

$$v_{Batt,k} = V_{OC}(SOC_k) - v_{TTC1,k} - v_{TTC2,k} - R_0 * i_{Batt}\,. \quad (11)$$

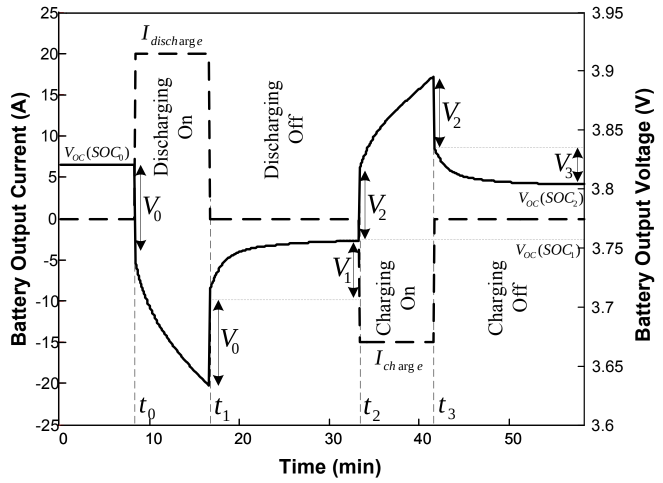

Fig. 2. Characteristic waveforms for battery output voltage and current during charging and discharging of lithium-ion cells.

III. ESTIMATION OF MODEL PARAMETERS

In this section the procedure of estimating the model parameters basing on battery measurements is demonstrated. In a first approach temperature and aging effects are neglected. The experimental parameter identification of the battery has been performed at the constant temperature of 25°C with relatively new and unused cells. The temperature and aging effects will be taken into account in a continuation of this work.

A. Charging and Discharging Process:

Fig. 2, shows characteristic curves of the battery output voltage and current when charging and discharging. In the following the different subintervals of the curves are described:

- Subinterval $S_0(t < t_0)$: In this subinterval the battery output current can be assumed to zero over a sufficient time, though the output voltage can reach the open circuit voltage value $V_{OC}(SOC_0)$, and while the output current is zero the SOC value is constant.
- Subinterval $S_1(t_0 \le t \le t_1)$: In this subinterval the battery is discharged with a constant current $I_{discharge} > 0$, first a steep decrease of the battery output voltage can be seen due to the internal resistance R_0, and then it continues to decrease exponentially controlled by the OCV (as the SOC is decreasing).
- Subinterval $S_2(t_1 \le t \le t_2)$: In this subinterval the battery output current $i_{Batt} = 0$, so the battery output voltage at first will have a steep increase due to R_0, and then it shows an exponential increase until it reaches $V_{OC}(SOC_1)$.
- Subinterval $S_3(t_2 \le t \le t_3)$: In this subinterval the battery is charged with a constant current $I_{charge} < 0$; at first a steep increase of the battery output voltage can be seen due to internal resistance R_0, and then it continues to increase exponentially controlled by the OCV (as the SOC is increasing).
- Subinterval $S_4(t \ge t_3)$: In this time subinterval the battery output current $i_{Batt} = 0$, so the battery output voltage at first will have a steep decrease due to R_0, and then it has an exponential decrease until it reaches $V_{OC}(SOC_2)$.

B. Ohmic Resistance:

The voltage drop across R_0 at the first time instant when charging (V_2) respectively discharging (V_0) can be taken to calculate R_0 [2], according to (12):

$$R_0 = \begin{cases} V_0 /(I_{\text{discharge}}): \text{for discharging} \\ -V_2 /(I_{\text{charge}}): \text{for charging} \end{cases} . \quad (12)$$

C. Estimation of OTC Model Parameters:

In this step battery output voltage measurements during the subintervals S_2 and S_4 are used, as in these subintervals OCV is constant, and the battery output voltage is just driven by the dynamic characteristics of the battery. The output voltage v_{Batt} during S_2 and S_4 can be calculated according to the OTC model by setting i_{Batt} to zero in (2) and (3), then solving the differential equation as shown in (13):

$$v_{Batt}(t) = \begin{cases} S_2 : V_{OC}(SOC_1) - v_{OTC}(t_1) * \exp(-t/\tau_{OTC}) \\ S_4 : V_{OC}(SOC_2) - v_{OTC}(t_3) * \exp(-t/\tau_{OTC}) \end{cases}, \quad (13)$$

where:

$$\tau_{OTC} = R_{OTC} * C_{OTC} . \quad (14)$$

The identification of OTC model parameters necessitates the estimation of the values $V_{OC}(SOC_1)$, $V_{OC}(SOC_2)$, $v_{OTC}(t_1)$, $v_{OTC}(t_3)$ and τ_{OTC} in (13) and (14). In order to estimate these parameters a nonlinear least squares algorithm is applied (nonlinear data fitting) to search for the values which lead to the best fit between the given measurements and the nonlinear function (in this case an exponential function $f(t) = A + B * \exp(-\alpha * t)$ is used).

The output of the nonlinear least squares algorithm is a vector of the coefficients A, B and α. These coefficients will be used to calculate the OTC model parameters through (15) to (19):

$$\begin{cases} S_2 : V_{OC}(SOC_1) = A, v_{OTC}(t_1) = B \\ S_4 : V_{OC}(SOC_2) = A, v_{OTC}(t_2) = B \end{cases}, \quad (15)$$

$$\tau_{OTC} = 1/(\alpha) , \quad (16)$$

$$\begin{cases} T_{\text{discharge}} = t_1 - t_0 \\ T_{\text{charge}} = t_3 - t_2 \end{cases}, \quad (17)$$

$$R_{OTC} = \begin{cases} S_2 : v_{OTC}(t_1) / \{[1 - \exp(-T_{\text{discharge}} / \tau_{OTC})] * I_{\text{discharge}}\} \\ S_4 : v_{OTC}(t_3) / \{[1 - \exp(-T_{\text{charge}} / \tau_{OTC})] * I_{\text{charge}}\} \end{cases}, \quad (18)$$

$$C_{OTC} = \tau_{OTC} / R_{OTC} . \quad (19)$$

D. Estimation of TTC Model Parameters:

TTC model parameters can be estimated the same way as for the OTC model, by taking into consideration the two RC networks instead of one in the OTC model. The TTC model output voltage can be expressed during the subintervals S_2 and S_4 by (20) and (21):

TABLE I
SLPB120216216 Cell Data

Typical Capacity		53 Ah
Nominal Voltage		3.7 V
Charge Condition	Max. Current	53A
	Voltage	4.2 ± 0.03 V
Discharge Condition	Continuous Current	159 A
	Peak Current	260 A
	Cut-off Voltage	3.0 V

$$v_{Batt} = \begin{cases} S_2 : V_{OC}(SOC_1) - v_{TTC1}(t_1) * \\ \qquad \exp(-t/\tau_{TTC1}) - v_{TTC2}(t_1) * \exp(-t/\tau_{TTC2}) \\ S_4 : V_{OC}(SOC_2) - v_{TTC1}(t_3) * \\ \qquad \exp(-t/\tau_{TTC1}) - v_{TTC2}(t_3) * \exp(-t/\tau_{TTC2}) \end{cases}, \quad (20)$$

where:

$$\begin{cases} \tau_{TTC1} = R_{TTC1} * C_{TTC1} \\ \tau_{TTC2} = R_{TTC2} * C_{TTC2} \end{cases} . \quad (21)$$

The identification of the TTC model requires to estimate the values $V_{OC}(SOC_1)$, $V_{OC}(SOC_2)$, $v_{TTC1}(t_1)$, $v_{TTC1}(t_3)$, $v_{TTC2}(t_1)$, $v_{TTC2}(t_3)$, τ_{TTC1} and τ_{TTC2} in (20) and (21). In this case an exponential function with two time constants $f(t) = A + B * \exp(-\alpha * t) + C * \exp(\beta * t)$ is used.

The output of the nonlinear least squares algorithm is a vector of the coefficients A, B, C, α, and β. These coefficients will be used to calculate all TTC model parameters through (22) to (27):

$$\begin{cases} S_2 : V_{OC}(SOC_1) = A \\ S_4 : V_{OC}(SOC_2) = A \end{cases}, \quad (22)$$

$$\begin{cases} S_2 : v_{TTC1}(t_1) = B, v_{TTC2}(t_1) = C \\ S_4 : v_{TTC1}(t_3) = B, v_{TTC2}(t_3) = C \end{cases}, \quad (23)$$

$$\begin{cases} \tau_{TTC1} = 1/\alpha \\ \tau_{TTC2} = 1/\beta \end{cases}, \quad (24)$$

$$R_{TTC1} = \begin{cases} S_2 : v_{TTC1}(t_1) / \{[1 - \exp(-T_{\text{discharge}} / \tau_{TTC1})] * I_{\text{discharge}}\} \\ S_4 : v_{TTC1}(t_3) / \{[1 - \exp(-T_{\text{charge}} / \tau_{TTC1})] * I_{\text{charge}}\} \end{cases}, \quad (25)$$

$$R_{TTC2} = \begin{cases} S_2 : v_{TTC2}(t_1) / \{[1 - \exp(-T_{\text{discharge}} / \tau_{TTC2})] * I_{\text{discharge}}\} \\ S_4 : v_{TTC2}(t_3) / \{[1 - \exp(-T_{\text{charge}} / \tau_{TTC2})] * I_{\text{charge}}\} \end{cases}, \quad (26)$$

$$C_{TTC1} = \begin{cases} \tau_{TTC1} / R_{TTC1} \\ \tau_{TTC2} / R_{TTC2} \end{cases} . \quad (27)$$

IV. Experimental and Computational Results

For the experimental tests and modeling lithium polymer battery cells from the manufacturer Kokam have been used. Some important cell data are depicted in Table I.

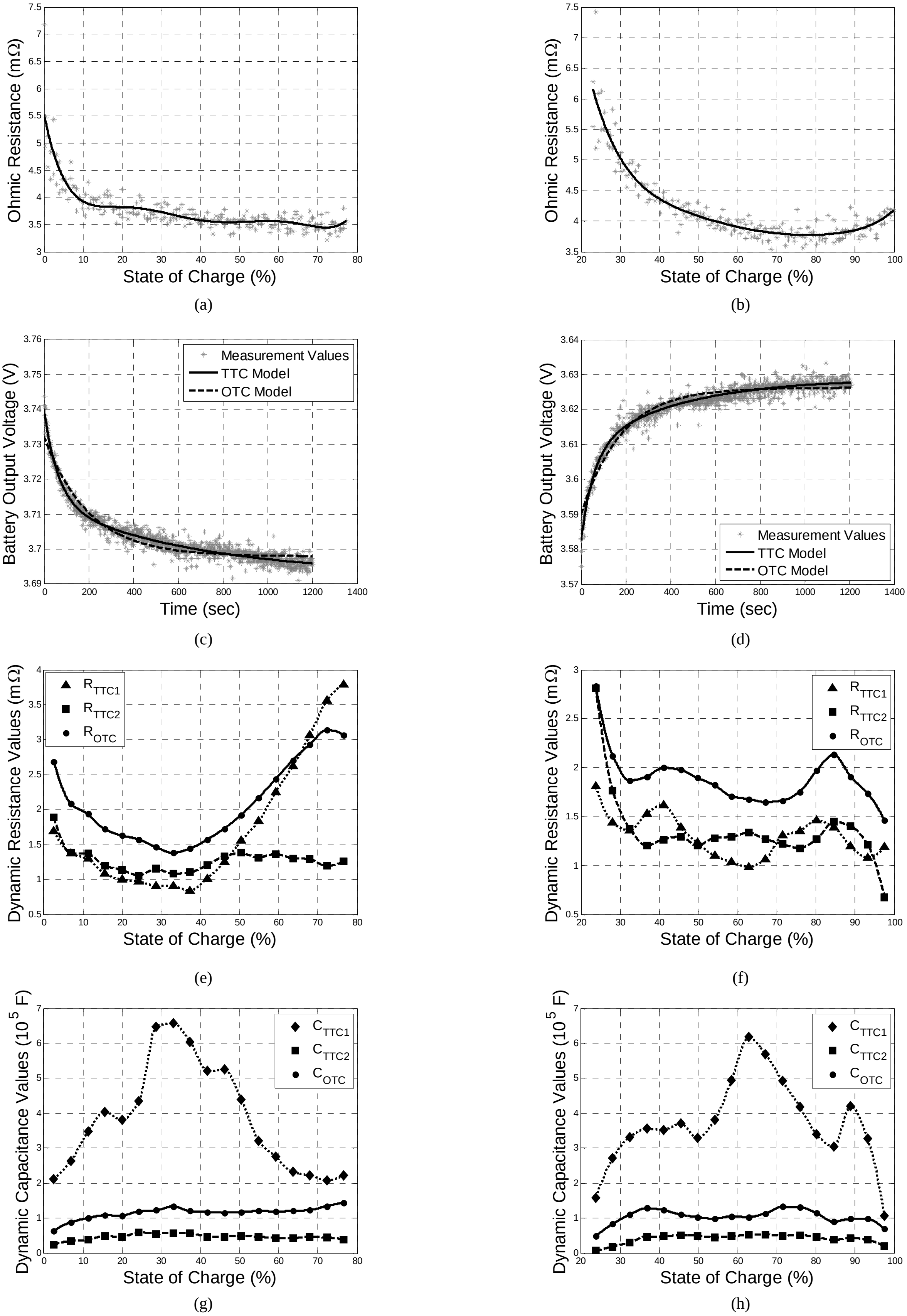

Fig. 3. (a), (b): ohmic resistance R_0 as a function of SOC during charging and discharging processes respectively; (c), (d): output voltage measurement and computational results during charging and discharging processes respectively; (e), (f): dynamic resistance values as a function of SOC during charging and discharging processes respectively; (g), (h): dynamic capacitance values as a function of SOC during charging and discharging processes respectively.

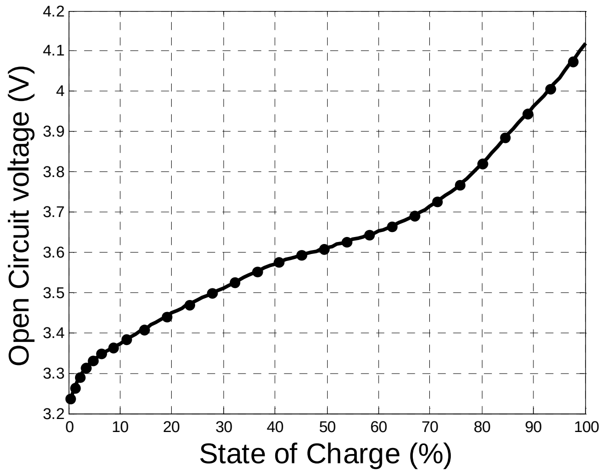

Fig. 4. Battery open circuit voltage as a function of SOC.

To identify the model parameters, a battery test bench was set up. In this test bench a current signal with rectangular shape has to be applied to the battery with short and long interrupts. At the same time the battery output voltage has to be measured. The ohmic resistance R_0 of the battery can be calculated during short interrupts of the current signal, while OTC and TTC model parameters need to be estimated during long interrupts.

A. Ohmic Resistance Results:

Ohmic resistance R_0 results are shown in Fig. 3a and 3b for charging and discharging processes respectively. It can be seen that R_0 increases at low SOC values in both, charging and discharging processes.

B. Values of the RC-Network Elements:

Fig. 3c and 3d. depict the battery output voltage measurement with the OTC and TTC model values for one long interrupt during charging and discharging processes respectively at 25°C. This processes are repeated over all long interrupts. From these two figures it can be seen that the TTC model has a better fit to the measurements than the OTC model, therefore the TTC model gives a better representation of the battery dynamics compared to the OTC model.

Dynamic resistance values are shown in Fig. 3e and 3f in both, charging and discharging processes respectively. These two figures demonstrate that there is not such a big deviation in the dynamic resistance values as it is the case with the dynamic capacitances. Dynamic capacitance values are depicted in Fig. 3g and 3h during both, charging and discharging processes respectively.

In these figures it can be seen that C_{TTC1} has a value being about 10 times greater than C_{TTC2}, that leads to a greater time constant τ_{TTC1} which is responsible for the long term effects in the battery. Fig. 4 shows the battery open circuit voltage in dependence of SOC.

V. CONCLUSION

This paper presents three different equivalent circuit diagrams for lithium-ion batteries. The IR model is a very simple model, but it does not represent at all the dynamics of the battery and therefore it is not suitable for an accurate SOC determination during any dynamical operation. The OTC model describes the dynamic characteristics of the battery approximately (Fig. 3c and 3d). By adding a second RC network the dynamics of the lithium-ion battery can be approximated very accurately (Fig. 3c and 3d), thus a good estimation of SOC can be expected.

This paper demonstrates how to identify the parameters of the equivalent circuit diagrams of lithium-ion cells from characteristic measurements. The next step of this research consists of the application of an Extended Kalman Filter to obtain the optimum estimation of SOC. A comparison of the results with the Ampere-counting method will be shown.

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