Optimal period finding of service of railway transport automatic systems

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Abstract
The service strategy of railway transport automatic systems which is used on Latvian railway is described in this paper. 2 main values which show the functionality quality of automatic systems are found out. Two methods of finding out optimal period of doing schedule services are offered – Newton-Rawson and Given-Find metrology of MatchCAD mathematical program. Optimal period of doing schedule services is calculated for every criteria – ready coefficient and middle summary expends in unit time by using offered methods. Functionality quality values of automatic system which has 15 different elements is calculated for found out optimal periods of schedule services.

Keywords
railway, automatic system, safety, schedule service, optimal periodicity, ready coefficient

Introduction
Safety of transportation process on railway transport is depended from train regulating system safety in big manner.
The problem of automatic block (AB), electrical interlocking (EI) and automatic locomotive signalization (ALSN) safety providing problem was always actual problem, so it is necessary to prevent their faults in time.
Combination of two service strategies – of prophylactic one and reductive one is widely used for safety increasing of automatic technical system functionality on Latvian railway.
The period of doing schedule services must be precisely planed and sudden faults must be corrected in time for maintenance personnel effective usage and for good organization of work and for performance of planed values.

Maintenance process
If do analyzes of used service strategies of automatic systems on Latvian railway more closer then order of using of multiply elements of such systems will be as described below:
the prophylactic doing moment is found out in the beginning of some system using when concrete set operating time is;
if there is fault in system before moment of doing planed prophylactic, then emergency repair of fault element is done. System is switched off during the emergency repair, so element safety characteristics is not decreased and system’s operating time is not increasing;
when set value is achieved, then planed prophylactic is done not depending from another factors, including the number of emergency repairs during given period. System is fully renewed when planed prophylactic is done;
when prophylactic work is done, new system’s operating time is found out for the next period.
When systems work time is near the set operating time, then next planed prophylactic is done and all maintenance process repeats.

Criteria of optimization
Optimal time $\tau_0$ of doing schedule services can be different for different criteria and, of course, the maximum value of criteria is when optimal $\tau_0$ is found out for this criteria.
There is many criteria of optimization of $\tau_0$, but the basic criteria and of course the most important of railway transport automatic systems can be: ready coefficient $K_g$. $K_g$ it is integrated generalized safety value, which describes equipment reliability and recoverability. In principle $K_g$ shows system functioning probability in time moment $t$;
middle total operation costs per hour $C_g$. $C_g$ dimension – [units/hour]. This parameter is very convenient for budget calculation, because it can show total expenses for any period of time in future.

Fault influence on characterize values in complex systems
All railway transport automatic systems – it is complex systems, i. e. they consist of many different elements $m$ and all of them have individual parameters (fault rate $\lambda$, cumulative distribution function of no failure operating time $F(t)$). How to find the optimal period of prophylactic works for maintenance process described above for such systems?
It is necessary to examine which processes influence to the value of $\tau_0$, which will be found out by described above two criteria.

Fig.1 shows the main factors which are moving $\tau_0$ in increasing or decreasing side when $K_g$ value is optimized.

![Diagram](TP-TAT_0-t)

Fig. 1. Factors which have influence on $\tau_0$ value, when $K_g$ value is optimized.

On Fig. 1: $TP$ - duration time of prophylactic work; $TAT_i$ - time which is necessary to renew functionality of faulted $i^{th}$ element, it sums together time which is necessary for fault finding and time which is necessary for faulted element replace. In that case, when doing the prophylactic service the system cannot do their functions during time $TP$, then it possible to suppose that than $\tau_0$ is bigger, then less number of prophylactics have to be done, and correspondingly bigger $K_g$ of all system is.

But if the value of $\tau_0$ is increasing, then probability $Q(t)$ of fault of automatic system is increasing too. So if $Q(t)$ is increasing then more and more faulted elements have to be replaced and more economical penalties $c_j$ are in system. $c_j$ - it is economical penalties which are taking place for every element. System doesn’t do their functions during element work renewing and, of course, it $C_s$ is increasing.

Indexes of system finding

For $\tau_0$ finding taking in to account factors which have influence to it value Fig.1 during $K_g$ value optimization expression (1) must be used. The root of equation (1) – it is optimal period of doing service works by $K_g$ criteria for described above maintenance process.

$$\tau \sum_{i=1}^{m} TAT_i HP_i(t) - \sum_{i=1}^{m} TAT_i H_i(t) - TP = 0$$

where:
- $H_i(t)$ - renewal function of $i^{th}$ element;
- $HP_i(t)$ - differential from renewal function of $i^{th}$ element;
- $m$ - number of elements in system.

$K_g$ of system for found $\tau_0$ can be found out from by using formula (2).

$$K_g = \frac{1}{1 + \sum_{i=1}^{m} TAT_i HP_i(\tau_0)}$$

For $\tau_0$ finding taking in to account factors which have influence to it value Fig.2 during $C_s$ value optimization expression (3) must be used. The root of equation (3) – it is optimal period of doing service works by $C_s$ criteria for described above maintenance process.

$$\tau \sum_{i=1}^{m} c_i TAT_i HP_i(t) - \sum_{i=1}^{m} c_i TAT_i H_i(t) - cp TP = 0$$

$K_g$ of system for found $\tau_0$ can be found out from by using (4) formula.

$$C_s = \sum_{i=1}^{m} c_i TAT_i HP_i(\tau)$$
Renewal function of \( i \)th element can be found out by using Laplace transformation of element cumulative distribution function of no failure operating time \( F(t) \) as it shown in (5) formula. After inverse Laplace transformation \( H_i(t) \) can be found.

\[
L[H_i(s)] = \frac{L(F(s))}{1 - L(F(s))}
\]  

(5)

**Table 1. Some initial conditions**

<table>
<thead>
<tr>
<th>Nr.</th>
<th>( n )</th>
<th>( F(t) )</th>
<th>( \lambda, 1/\text{h} )</th>
<th>( TAT_i, \text{h} )</th>
<th>( TP, \text{h} )</th>
<th>( c_i ), unit/h</th>
<th>( cp ), unit/h</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>( 1 - e^{-\lambda_1 t} )</td>
<td>( 3 \times 10^{-3} )</td>
<td>1</td>
<td>1</td>
<td>3000</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>( 1 - e^{-\lambda_2 t} )</td>
<td>( 4 \times 10^{-8} )</td>
<td>2</td>
<td>2</td>
<td>3000</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>( 1 - e^{-\lambda_3 t} )</td>
<td>( 5 \times 10^{-4} )</td>
<td>2</td>
<td>2</td>
<td>3000</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>( 1 - e^{-\lambda_4 t} )</td>
<td>( 3 \times 10^{-4} )</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Mathematical program MatchCAD can be used for finding optimal values of systems functionality quality. Renewal function of elements:

\[
d = \sqrt{-\lambda(4 + \lambda)}
\]

\[
H_1(t) = -2e^{-\frac{\lambda}{2} t} \left( 1 - \sum_{i=1}^{n} \frac{(x_i + \lambda t)^{n-1}}{(n-1)!} \right) d
\]

Differential from renewal function:

\[
HP_1(t) = e^{\frac{\lambda}{2} t} \left( \lambda \cdot \sin \left( \frac{1}{2} d \cdot t \right) - \cos \left( \frac{1}{2} d \cdot t \right) \right) d
\]

Prophylactic optimal period \( \tau_0 \) by \( K_g \) criteria is offered to find by using Newton-Rawson method:

matrix\( H_1(t) = (H_1(t), H_1(t)) \)

matrix\( H_2(t) = (H_1(t), H_2(t), H_1(t), H_2(t, H_1(t)) \)

matrix\( H_3(t) = (H_2(t), H_3(t)) \)

matrix\( H_4(t) = \operatorname{augment}(\text{matrix}\ H_1(t), \text{matrix}\ H_2(t), \text{matrix}\ H_3(t)) \)

matrix\( HP_1(t) = (HP_1(t), HP_1(t), HP_1(t), HP_1(t)) \)

matrix\( HP_2(t) = (HP_1(t), HP_1(t), HP_2(t), HP_1(t)) \)

matrix\( HP_3(t) = (HP_2(t), HP_P(t), HP_3(t), HP_P(t)) \)

matrix\( HP_4(t) = \operatorname{augment}(\text{matrix}\ HP_1(t), \text{matrix}\ HP_2(t), \text{matrix}\ HP_3(t)) \)

**Optimal period of prophylactic by \( K_g \) criteria is:**

\[ \xi := 0.001 \]

\[
y_1(t) = \tau \cdot \sum_{i=1}^{m} \text{matrix} \tau_1[i] \cdot \text{matrix} \tau H_1[i,1] \]

\[
y_2(t) = \sum_{i=1}^{m} \text{matrix} \tau_1[i] \cdot \text{matrix} \tau H_1[i,1] - \text{TP} \]

\[
y(t) = y_1(t) + y_2(t) \]

**Example**

Let some system is given which consists from 15 elements. Such elements are grouped in to four groups with \( n \) elements in each group. For all elements the same \( F(t) \) is given. It is accepted, that system works without interruptions. All parameters are given in table 1.

\[
\begin{aligned}
\text{Days} &= \frac{x}{24} \\
\tau_0 \text{ value} &= 2679 \text{ hours or 112 days by } K_g \text{ criteria.} \\
\end{aligned}
\]

Second parameter - \( \tau_0 \) by \( C_g \) criteria is offered to find by using MatchCAD program’s block – Given-Find:

Initial condition

\[
\tau := 500
\]

Given

\[
\tau \sum_{i=1}^{m} \text{matrix} Q_1[i] \cdot \text{matrix} \tau T[1,i] \cdot \text{matrix} H_1[1,i] - \text{...} = 0
\]

\[
+ \sum_{i=1}^{m} \text{matrix} Q_1[i] \cdot \text{matrix} \tau T[1,i] \cdot \text{matrix} H_1[1,i] - cp \cdot \text{TP} = 0
\]

\[
x = \text{Find}(\tau) \quad x = 1291.8162016652 \quad \tau = x
\]

\[
\text{Days} = \frac{x}{24} \quad \text{days} = 54
\]

\( \tau_0 \) value is 1292 hours or 54 days by \( K_g \) criteria.
Two offered below methods for $\tau_0$ finding are identical and it is possible to use any of them. The value of selected criteria for founded $\tau_0$ by two different criteria is calculated by using formulas (2) and (4):

\[
K_g(\tau) = 0.9995131066
\]
\[
C_s(\tau) = 112.4690676658
\]

**Conclusions**

- 2 offered methods of finding $\tau_0$ are giving identical result and they are easy in use;
- $\tau_0$ value depends from selected criteria;
- $\tau_0$ value can be strongly different depending from selected criteria (in viewed example – in 2 times).

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**References**
